Int. J. Heat Mass Transfer. Vol. 21, pp. 477–480. Pergamon Press 1978. Printed in Great Britain

LAMINAR CONVECTION WITH RADIATION: EXPERIMENTAL AND THEORETICAL RESULTS

R. GREIF
Department of Mechanical Engineering, University of California, Berkeley, CA 94720, U.S.A.

(*Received* 17 *February* 1977 *and in revised form* 9 *August* 1977)

Abstract—Temperature profiles have been measured for laminar convection of carbon dioxide in a forced upflow. The data are in good agreement with a simplified analysis which includes the effects of buoyancy and thermal radiation. Generalized results have also been obtained in terms of the basic parameters.

NOMENCLATURE

a, constant equal to 1.0;
A, total band absorptance;
A_0, band width parameter;
b, constant equal to 1.25;
$B_{\omega c}$, Planck's function;
c_p, specific heat;
C_0, correlation parameter;
f_2, pressure broadening parameter;
g, gravitational constant;
H, radiation–conduction parameter;
h, heat-transfer coefficient, $q_w 2R/K(T_w - T_b)$;
K, thermal conductivity;
P, pressure;
q_w, wall heat flux;
r, radial coordinate;
$\bar{r}$, dimensionless radial coordinate, r/R;
R, pipe radius;
Ra, Rayleigh number, $\beta g \tau R^3/\alpha \nu$;
$\bar{S}$, dimensionless coordinate in radiation flux term;
T, temperature;
T_w, wall temperature;
u, velocity;
$\bar{u}$, dimensionless velocity, $uR/\alpha\gamma$;
x, axial coordinate.

Greek symbols

α, thermal diffusivity;
β, coefficient of thermal expansion;
γ, angle between line of sight and radial direction r;
γ, pressure drop parameter, $R^3/\alpha\nu\left[\dfrac{1}{\rho_w}\dfrac{dP}{dx}+g\right]$;
$\bar{\theta}$, dimensionless temperature, $(T_w - T)/\tau\gamma$;
ρ, density;
τ, temperature gradient, $(dT_w/dx)R$.

Subscripts

b, bulk value;
c, convection;
r, radiation;
w, wall value;
ωc, evaluated at band center.

Superscript

$'$, derivative.

INTRODUCTION

IN A PREVIOUS study measurements were made of the heat transfer to transparent gases flowing in a long vertical heated tube [1]. Two theoretical solutions were also obtained; one based on the complete conservation equations which included such effects as variable properties, flow and thermal development, dissipation, etc., and the other based on a simplified, fully developed flow with constant properties except for the density variation in the body force term. Results were obtained for air and argon over ranges of the Reynolds number from 1850 to 2100, for Rayleigh numbers that varied from 70 to 80. The simplified theoretical results were in very good agreement with both the more complete numerical calculations and with the measured temperature profiles.

The present work is an extension of [1] and investigates the heat transfer to absorbing and emitting gases. The simplified fully developed formulation is extended to include thermal radiation effects and the results obtained from this method are then compared with the experimental data for carbon dioxide. Generalized results have also been obtained in terms of the basic parameters.

LAMINAR CONVECTION WITH RADIATION

Measurements were made with carbon dioxide flowing in an inconel steel tube, 6.7 m long, 0.124 cm thick and 5.08 cm O.D. The tube was supported vertically and heated by passing an electric current from a 440 V 3 phase AC power supply through electrodes attached to both ends. The tube was surrounded by radiation shields and maintained under vacuum to reduce convection losses. A thermocouple probe, inserted from the upper end to a location 108 diameters from entry, permitted radial temperature traverses to be made. Detailed descriptions of the system and the probe are available [1, 2] and will not be repeated here.

On the basis of the previous study with air and argon the fully developed momentum and energy equations were used as the starting point for the analysis:

$$0 = -\frac{dP}{dx} - \rho g + \mu \frac{1}{r}\frac{d}{dr}\left(r\frac{\partial u}{\partial r}\right) \tag{1}$$

$$u\frac{\partial T}{\partial x} = \alpha \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{1}{r\rho c_p}\frac{\partial}{\partial r}(rq_R). \tag{2}$$

The determination of the radiative flux in cylindrical coordinates has been studied by many investigators [3–10] and these analyses have been reviewed and extended by Donovan [11] with particular attention devoted to large optical depths. Omitting the details the resulting equations may be written in the following dimensionless form ([11] cf. Nomenclature):

$$\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\frac{d\bar u}{d\bar r}\right) = -1 + Ra\bar\theta \tag{3}$$

$$\bar u + \frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\frac{d\bar\theta}{d\bar r}\right) = -\frac{R}{K}\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\left\langle 4a\int_0^{\pi/2}\cos\gamma\left\{R\int_0^{\sqrt{(1-\bar r^2\sin^2\gamma)}}(-B'_{\omega c}|_{T_w}\bar\theta)A'[bR(\bar S+\bar r\cos\gamma)]\,d\bar S\right.\right.\right.$$
$$\left.\left.\left. -R\int_{\bar r\cos\gamma}^{\sqrt{(1-\bar r^2\sin^2\gamma)}}(-B'_{\omega c}|_{T_w}\bar\theta)A'[bR(\bar S-\bar r\cos\gamma)]\,d\bar S + R\int_0^{\bar r\cos\gamma}(-B'_{\omega c}|_{T_w}\bar\theta)A'[bR(-\bar S+\bar r\cos\gamma)]\,d\bar S\right\}d\gamma\right\rangle\right). \tag{4}$$

It is noted that the temperature differences are small so that the density and spectral blackbody functions have been linearized according to:

$$\rho = \rho_w[1+\beta(T_w-T)] \tag{5}$$

$$B_{\omega c} = B_{\omega c}(T_w) + B'_{\omega c}(T_w)[T_w - T]. \tag{6}$$

To proceed with the solution of the coupled equations (3) and (4) the band absorptance (or its derivative) must be specified. The relation used is given by (cf. Edwards and Menard [12], Tien and Lowder [13])

$$A = A_0 \ln[1+f_2u] \tag{7}$$

which then leads to the following result for the energy equation:

$$\bar u = -\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\frac{d\bar\theta}{d\bar r}\right) + H\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\left\langle 4a\int_0^{\pi/2}\left\{+f_2u_0\int_0^{\sqrt{(1-\bar r^2\sin^2\gamma)}}\frac{\bar\theta\,d\bar S}{f_2u_0b(\bar S+\bar r\cos\gamma)+1}\right.\right.\right.$$
$$\left.\left.\left. -f_2u_0\int_{\bar r\cos\gamma}^{\sqrt{(1-\bar r^2\sin^2\gamma)}}\frac{\bar\theta\,d\bar S}{f_2u_0b(\bar S-\bar r\cos\gamma)+1} + f_2u_0\int_0^{\bar r\cos\gamma}\frac{\bar\theta\,d\bar S}{f_2u_0b(-\bar S+\bar r\cos\gamma)+1}\right\}\cos\gamma\,d\gamma\right\rangle\right) \tag{8}$$

where $H = RB'_{\omega c}(T_w)A_0/K$ and $u_0 = \rho C_0^2 R$. Calculations carried out showed that the 15 µm vibrational–rotational band of carbon dioxide was the only band which made a significant contribution to the radiation transport. The correlation parameters for this band are specified by Edwards and Menard [12] and Tien [14] (cf. Donovan [11]). Values of $a = 1.0$ and $b = 1.25$ are specified based on the work in [6, 8–10]. Equations (3) and (8) were solved by the method of collocation [11, 15].

RESULTS AND DISCUSSION

Figure 1 displays the experimental data for the radial temperature profiles in carbon dioxide along with three theoretical results: (1) pure forced convection, (2) combined convection, that is, forced and natural convection and (3) combined convection in an absorbing and emitting gas. The effect of buoyancy on transparent gases, e.g. air and argon, has been discussed [1]. Briefly, it was shown that the radial temperature profiles and the wall temperature variations resulting from a simplified, fully developed analysis were in excellent agreement with more complete numerical calculations and with experimentally measured values. On the basis of these results, fully developed calculations were carried out for the problem of forced convection with buoyancy and radiation interactions [cf. equations (3) and (8)].

The effect of radiation is to increase the energy

Table 1. Summary of heat-transfer results for carbon dioxide

Run	Re	$Ra^{1/4}$	$(hD/K)_{108D}$	$(h_cD/K)_{108D}$	$(h_{rad}D/K)_{108D}$	H	f_2u_0	γ	$T_w(K)$
183	1910	3.83	7.09	6.37	0.72	0.056	22.9	31 000	321
190	1972	3.76	6.97	6.26	0.71	0.056	22.8	30 000	322

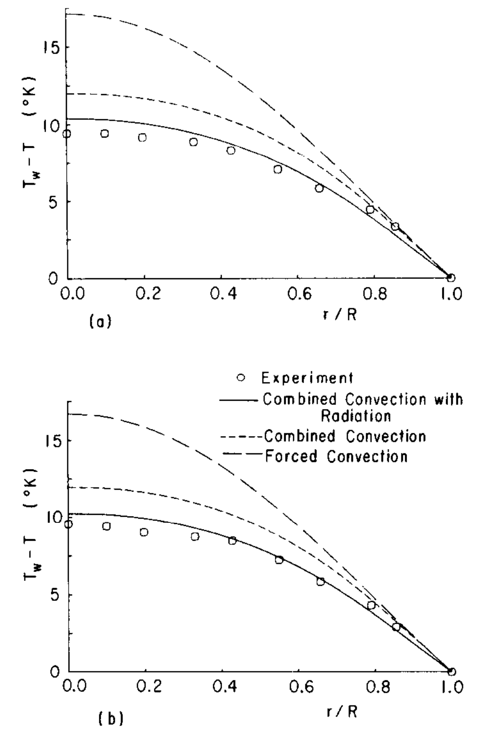

FIG. 1. Experimental and theoretical temperature profiles for carbon dioxide. (a) Run 183; (b) Run 190.

transport, thereby decreasing the temperature difference between the gas and the wall (cf. Fig. 1). The general agreement between the data and theoretical predictions is good although there is some discrepancy. However, in view of the approximations that have been made in determining the theoretical results, e.g. linearization of the spectral blackbody function, one-dimensional radiation transport, absorptance correlation, fully developed conditions, constant properties, etc., it is felt that the agreement is satisfactory. The results for the heat transfer are summarized in Table 1 for carbon dioxide. For the limited range of conditions tested, it is seen that the "radiation heat-transfer coefficient", h_{rad}, is 10% of the total heat-transfer coefficient, h.

Theoretical calculations have also been carried out over the following range of parameters:

Radiation–conduction parameter: $H = 0.0, 0.05, 0.10$

Modified optical depth: $f_2 u_0 = 15.0, 30.0, 45.0, 60.0$

Rayleigh number: $(Ra)^{1/4} = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0$.

Since a detailed presentation and discussion of these results are available [11] only a brief coverage will be given here. Also, the planar problem is presented in [16].

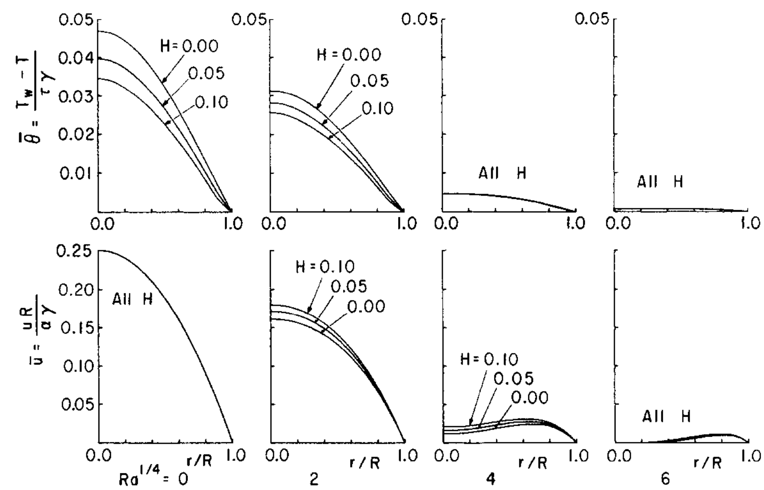

FIG. 2. Theoretical dimensionless temperature and velocity profiles for $f_2 u_0 = 45$.

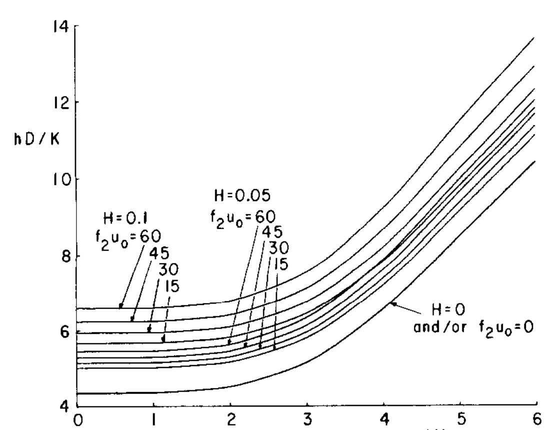

FIG. 3. Theoretical heat-transfer results.

In Fig. 2 dimensionless temperature and velocity profiles are given for a modified optical depth, $f_2 u_0$, of 45. The first frames, upper and lower, correspond to the non-buoyant problem, $Ra = 0$. As the radiation transport increases, i.e. as H increases, the temperature difference decreases (as noted above) which results in the flatter profiles shown. For the non-buoyant case the velocity is unaffected by the radiation transport since the momentum and energy equations are uncoupled.

Note that the effect of buoyancy ($Ra \neq 0$) is to decrease the velocity in a tube in a heated upflow. Therefore, the effect of radiation to decrease temperature differences reduces the influence of natural convection. Thus, the velocities for the radiating natural convection case are greater than those resulting from the corresponding non-radiating case (cf. Fig. 2). Results for the heat transfer are presented in Fig. 3 and details for the radiation and convection contributions, volume flow rates, and bulk temperatures are given in [11].

Acknowledgement—The author acknowledges with appreciation the support of this research by the National Science Foundation.

REFERENCES

1. R. Greif, An experimental and theoretical study of heat transfer in vertical tube flows, *J. Heat Transfer*, to be published.
2. I. S. Habib and R. Greif, Heat transfer to a flowing nongray radiating gas: an experimental and theoretical study, *Int. J. Heat Mass Transfer* **13**, 1571–1582 (1970).
3. W. Geffecken, Radiale Strahlungsleitung in langer Zylindern, *Berichte* **32K**(VII), 23–39 (1959).
4. Y. S. Kuznetsov, Temperature distribution in an infinite cylinder and in a sphere in a state of non-monochromatic radiation equilibrium (English translation), in *U.S.S.R. Comp. Math. Math. Phys.* **2**, 230–254 (1963).
5. A. S. Kesten, Radiant heat flux distribution in a cylindrically symmetric nonisothermal gas with temperature dependent absorption coefficient, *J. Quantve Spectros. Radiat. Transfer* **8**, 419–434 (1968).
6. I. S. Habib and R. Greif, Nongray radiative transport in a cylindrical medium, *J. Heat Transfer* **92**, 28–32 (1970).
7. A. G. Guy, Pilkington Research and Development Laboratories, Lancashire, England, Personal Communication.
8. Z. Chiba and R. Greif, Discussion of "Nongray radiative transport in a cylindrical medium", *J. Heat Transfer* **95C**(1), 142 (Feb. 1973).
9. D. K. Edwards and A. T. Wassel, The radial radiative heat flux in a cylinder, *J. Heat Transfer* **95C**, 276–277 (1973).
10. A. T. Wassel and D. K. Edwards, Molecular gas radiation in a laminar or turbulent pipe flow, *J. Heat Transfer* **98C**, 101–107 (1976).
11. T. E. Donovan, Heat transfer in internal flows including buoyancy and thermal radiation, Ph.D. Dissertation, University of California, Berkeley, California (December 1975).
12. D. K. Edwards and W. A. Menard, Comparison of models for correlation of total band absorption, *Appl. Optics* **3**, 621–625 (1964).
13. C. L. Tien and J. E. Lowder, A correlation for total band absorptance of radiating gases, *Int. J. Heat Mass Transfer* **9**, 698–701 (1966).
14. C. L. Tien, Thermal radiation properties of gases, in *Advances in Heat Transfer*, Vol. 5, pp. 254–324. Academic Press, New York (1968).
15. L. Collatz, *The Numerical Treatment of Partial Differential Equations*, *3rd edn*, pp. 29, 409–413. Springer, Berlin (1960).
16. T. E. Donovan and R. Greif, Laminar convection with an absorbing and emitting gas, *Appl. Scient. Res.* **31**, 110–122 (1975).

CONVECTION LAMINAIRE AVEC RAYONNEMENT: RESULTATS THEORIQUES ET EXPERIMENTAUX

Résumé—Des profils de température sont déterminés pour la convection laminaire du gaz carbonique dans un mouvement forcé ascendant. Les mesures sont en bon accord avec une analyse simplifiée qui inclut les effets de gravité et de rayonnement thermique. Des résultats généralisés ont été aussi obtenus en fonction des paramètres fondamentaux.

LAMINARE KONVEKTION MIT RADIATION: EXPERIMENTELLE UND THEORETISCHE ERGEBNISSE

Zusammenfassung—Es wurden Temperaturprofile bei erzwungener laminarer aufsteigender Konvektion von Kohlendioxid gemessen. Die Meßergebnisse stimmen gut mit einer vereinfachten Analyse überein, welche die Einwirkungen des Auftriebs und der thermischen Strahlung berücksichtigt. Es wurden ebenfalls allgemeine Ergebnisse in Abhängigkeit von den Grundparametern erzielt.

ЛАМИНАРНАЯ КОНВЕКЦИЯ ПРИ НАЛИЧИИ ИЗЛУЧЕНИЯ. ЭКСПЕРИМЕНТАЛЬНЫЕ И ТЕОРЕТИЧЕСКИЕ РЕЗУЛЬТАТЫ

Аннотация — Проведено измерение профилей температуры при ламинарном течении двуокиси углерода в вынужденном восходящем потоке. Данные измерений хорошо согласуются с результатами упрощенного анализа, в котором учитываются эффекты свободной конвекции и теплового излучения. Полученные результаты обобщены с помощью основных параметров.